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## DEPARTMENTS.

## SOLUTIONS OF PROBLEMS.

## ALGEBRA.

253. Proposed by PROF. R. D. CARMICHAEL, Anniston, Ala.

Prove that  $x^5 + ax + b = 0$  is solvable by radicals if  $b = ma$ ,  $m$  being the negative of half the sum of any two roots of the original equation. Exhibit the solution.

Solution by DR. L. E. DICKSON, The University of Chicago.

Let  $r$  and  $s$  be the two roots for which  $m = -\frac{1}{2}(r+s)$ . Then

$$r^5 + ar + ma = 0, \quad s^5 + as + ma = 0.$$

Adding, we get  $r^5 + s^5 = 0$ . We exclude the trivial case  $m = 0$ , so that  $r + s \neq 0$ ,  $r = -es$ ,  $e$  being an imaginary fifth root of unity. From  $r + es = 0$ ,  $r + s = -2m$ , we get

$$(1) \quad s = \frac{2m}{e-1}, \quad r = \frac{-2me}{e-1}.$$

Now  $s$  will indeed be a root if and only if

$$32m^4 + al = 0, \quad l = (e+1)(e-1)^4.$$

Since  $e^4 + e^3 + e^2 + e + 1 = 0$ , we get  $l = 5(e^3 + e^2 + 1)$ . Now  $e^2 + e^3$  and  $e + e^4$  are the roots  $\frac{1}{2}(-1 \pm \sqrt{5})$  of  $z^2 + z - 1 = 0$ . Hence the problem stated is possible if and only if

$$64m^4 + 5(1 \pm \sqrt{5})a = 0, \quad a = \frac{4}{5}(1 \mp \sqrt{5})m^4.$$

For  $m$  arbitrary, and for this value of  $a$ , the equation  $x^5 + ax + ma = 0$  has the two roots (1); the remaining roots may be found by solving a cubic.

284. Proposed by DR. E. H. MOORE, The University of Chicago, Chicago, Ill.

Discuss the system of equations:

$$\begin{cases} x^k + y^l = a_k \\ x^l + y^k = a_l \end{cases} \quad (k, l \text{ distinct positive integers})$$

in general and for particular values of  $(k, l; a_k, a_l)$ .